CES, Logit, and Rational Inattention

Andrei Matveenko

CERGE-EI, Prague
amatveen@cerge-ei.cz

Poznan, July, 2016
A lot of easily accessible information is ignored

EXAMPLES:

- What is exchange rate EUR/PLN?
- Left digit bias (millage on odometer) in the used car market
- Often prices which are ending on .95 or .99 are perceived as the same

POSSIBLE REASON:
Information is costly to process
Information Theory and Rational Inattention

- Shannon, 1948
- Sims, 2003

"...adding information-processing constraints to the kind of dynamic programming problem that is used to model behavior in many current macroeconomic models. It turns out that doing so alters the behavior implied by these models in ways that seem to accord, along several dimensions, with observed macroeconomic behavior.”
This paper

- Rational Inattention techniques to model consumer behavior
- Implications for Industrial Organization and International Trade:
  - New microfoundation for CES demand system;
  - New connection between logit and CES utility function
Logit discrete choice model of a singular consumer leads to:

\[ P_i(v) = \frac{e^{v_i/\mu}}{\sum_{j=1}^{N} e^{v_j/\mu}} \]

CES utility function model of fictitious representative consumer:

\[ U = \left( \sum_{j=1}^{N} \beta_j q_j^\rho \right)^{1/\rho} \]

It is important to connect them for welfare analyses;
A known connection between Logit and CES

Anderson, De Palma, Thisse (1987)
Random Utility Model

\[ U_i = v_i + \mu \epsilon_i \]

heterogeneity and particular distribution of tastes (i.i.d. Gumbel)

\[ \mu = \frac{1 - \rho}{\rho} \]

Weaknesses:

- Why Gumbel?
- How \( \mu \) can change?
- What are \( \beta_j \)? (Anderson et al. just make them equal to 1)
In this paper

In our case:

- Agents are rationally inattentive
- no taste heterogeneity; information costs;
- Parameters of the CES \((\rho, \beta_j)\) – parameters of the Rational Inattention model
1) The DM chooses among N possible actions, is endowed with prior about their payoffs,
2) chooses what additional information to process,
3) receives signals of the selected form,
4) selects one action.

- Action set: $A = \{1, \cdots , N\}$
- State: $\mathbf{v} \in \mathbb{R}^N$ where $v_i$ is the payoff of action $i \in A$
- The DM is a Bayesian expected utility maximizer
The agent's problem is to find an information processing strategy (modeled as a joint distribution between $v$ and the chosen product $i$) maximizing

$$\mathbb{E} v_i - \lambda \kappa,$$

(utility: expected payoff less cost of information).

Entropy-based cost function (reduction of uncertainty):

$$\hat{c}(F) \equiv \lambda \left( H(G) - E_s[H(F(\cdot|s))] \right),$$

where $\lambda \geq 0$, $H(B)$ is entropy of $B$, measure of uncertainty, e.g.

$$H(B) = - \sum_{k} P_k \log(P_k).$$
$N$ goods, $i = 1, \ldots, N$ – perfect substitutes

each good has price $p_i$ but it becomes perfectly known only at the moment of payment

utility function of decision maker $u_i = \ln q_i$,

income $y$

hence, indirect utility function $v_i = \ln \left( \frac{y}{p_i} \right)$
• Observing prices is costly
• DM is rationally inattentive

Matějka and McKay (2015) derive:

\[ P_i(v) = \frac{P_i^0 e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{N} P_j^0 e^{\frac{v_j}{\lambda}}}, \quad i = 1, \ldots, N. \]
Conditional expected demand:

\[ D_i = \frac{P_i^0 p_i^{\frac{1}{\lambda}} - 1}{\sum_{j=1}^{N} P_j^0 p_j^{\frac{1}{\lambda}}}, \quad i = 1, ..., N. \]

where \( P_i^0 \) – prior belief, \( p_i \) – price of good \( i \)

The same demand system from CES:

\[ U = \left( \sum_{j=1}^{N} \beta_j q_j^\rho \right)^{\frac{1}{\rho}}, \]

where \( \rho = 1/(\lambda + 1) \), and the coefficients \( \beta_j \) depend positively on the corresponding unconditional probabilities \( P_j^0 \).
Marginal cost of information $\lambda$, prior knowledge $P_i^0$

Elasticity of substitution:

$$\sigma = \frac{1}{1 - \rho} = \frac{1}{\lambda} + 1 > 1.$$ 

Weighting coefficients:

$$\beta_i = \gamma \left( P_i^0 \right)^{1 - \rho} = \gamma \left( P_i^0 \right)^{\frac{\lambda}{1 + \lambda}}$$
Implications

Marginal cost of information $\lambda$

- If $\lambda \to 0$ then $CES \sim \max$
- If $\lambda \to \infty$ then $CES \sim Cobb - Douglas$

  Representative consumer follows only the prior knowledge (distributes her income proportionally to $P_i^0$)
- If $\lambda \uparrow$ then mark-up $1 - \rho \uparrow$
We endogenize elasticity of substitution and coefficients of CES function.
We connect theory of rational inattention with popular neoclassical economics model.
Such connection is important for policy implications and empirical analysis.
Thank you for your attention