CES, Logit, and Rational Inattention

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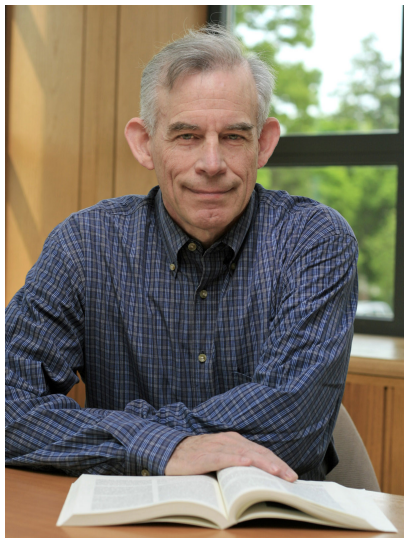
Attention & Behavior

- A lot of easily accessible information is ignored
  EXAMPLES:
- What is exchange rate EUR/PLN?
- Left digit bias (millage on odometer) in the used car market
- Often prices which are ending on .95 or .99 are perceived as the same

POSSIBLE REASON:
Information is costly to process

Information Theory and Rational Inattention

- Shannon, 1948
- Sims, 2003
- "...adding information-processing constraints to the kind of dynamic programming problem that is used to model behavior in many current macroeconomic models. It turns out that doing so alters the behavior implied by these models in ways that seem to accord, along several dimensions, with observed macroeconomic behavior."



This paper

- Rational Inattention techniques to model consumer behavior
- Implications for Industrial Organization and International Trade:
- New microfoundation for CES demand system;
- New connection between logit and CES utility function

Logit and CES

Logit discrete choice model of a singular consumer leads to:

$$P_i(v) = \frac{e^{v_i/\mu}}{\sum_{j=1}^{N} e^{v_j/\mu}}$$

CES utility function model of fictitious representative consumer:

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{1/\rho}$$

It is important to connect them for welfare analyses;

A known connection between Logit and CES

Anderson, De Palma, Thisse (1987)
Random Utility Model

$$U_i = v_i + \mu\varepsilon_i$$

heterogeneity and particular distribution of tastes (i.i.d. Gumbel)

$$\mu = \frac{1-\rho}{\rho}$$

Weaknesses:

- Why Gumbel?
- How μ can change?
- What are β_j? (Anderson et al. just make them equal to 1)

In this paper

In our case:

- Agents are rationally inattentive
- no taste heterogeneity; information costs;
- Parameters of the CES (ρ, β_j) – parameters of the Rational Inattention model

Problem of Rationally Inattentive DM 1

1) The DM chooses among N possible actions, is endowed with prior about their payoffs,
2) chooses what additional information to process,
3) receives signals of the selected form,
4) selects one action.

- Action set: $A = \{1, \cdots, N\}$
- State: $\mathbf{v} \in \mathbb{R}^N$ where v_i is the payoff of action $i \in A$
- The DM is a Bayesian expected utility maximizer

Problem of Rationally Inattentive DM 2

The agent's problem is to find an information processing strategy (modeled as a joint distribution between v and the chosen product i) maximizing

$$\mathbb{E}v_i - \lambda\kappa.$$

(utility: expected payoff less cost of information).
Entropy-based cost function (reduction of uncertainty):

$$\hat{c}(F) \equiv \lambda\Big(H(G) - E_{\mathbf{s}}[H(F(\cdot|\mathbf{s}))]\Big),$$

where $\lambda \geq 0$, $H(B)$ is entropy of B, measure of uncertainty, e.g.

$$H(B) = -\sum_k P_k \log(P_k).$$

Description of our Model

- N goods, $i = 1, ..., N$ – perfect substitutes
- each good has price p_i but it becomes perfectly known only at the moment of payment
- utility function of decision maker $u_i = \ln q_i$,
- income y
- hence, indirect utility function $v_i = \ln\left(\frac{y}{p_i}\right)$

RI

- Observing prices is costly
- DM is rationally inattentive

Matějka and McKay (2015) derive:

$$P_i(v) = \frac{P_i^0 e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{N} P_j^0 e^{\frac{v_j}{\lambda}}}, \quad i = 1, ..., N.$$

RI and CES 1

Conditional expected demand:

$$D_i = \frac{P_i^0 p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, ..., N.$$

where P_i^0 – prior belief, p_i – price of good i
The same demand system from CES:

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{\frac{1}{\rho}},$$

where $\rho = 1/(\lambda + 1)$, and the coefficients β_j depend positively on the corresponding unconditional probabilities P_j^0.

RI and CES 2

Marginal cost of information λ, prior knowledge P_i^0
Elasticity of substitution:

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1 > 1.$$

Weighting coefficients:

$$\beta_i = \gamma \left(P_i^0\right)^{1-\rho} = \gamma \left(P_i^0\right)^{\frac{\lambda}{1+\lambda}}$$

Implications

Marginal cost of information λ

- If $\lambda \to 0$ then $CES \sim \max$
- If $\lambda \to \infty$ then $CES \sim Cobb - Douglas$
  Representative consumer follows only the prior knowledge (distributes her income proportionally to P_i^0)
- If $\lambda \nearrow$ then mark-up $1 - \rho \nearrow$

Conclusion

- We endogenize elasticity of substitution and coefficients of CES function
- We connect theory of rational inattention with popular neoclassical economics model
- Such connection is important for policy implications and empirical analysis

Thank you for your attention