Loss-averse decision analysis in overbooking

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1.Introduction

Introduction

Many service industries, such as healthcare, hotel, and airline ticketing, share common characteristics:

- (1) The capacity is constrained and perishable;
- (2) Bookings are accepted for future use;
- (3) Customers are allowed to cancel bookings or not show at the time of service;
- (4) The cost of denying service to a customer with booking is relatively not too high.

Managers in these businesses accept reservations and subsequently runs the risk of cancellations and no-shows.

The strategy of overbooking capacity is commonly practiced.

Introduction

- Managing overbooking requires a method that gives consideration to both managers' attitude pertaining to the loss due to fine for bumped customers and the potential increase in capacity utilization.
- Traditional overbooking models are mainly based on the assumption that decision makers are rationally loss-neutral.
- There is an increasing body of behavioral evidence for loss-aversion, indicating that **changes for loss loom larger than equivalent changes for the gain.**

2. Model Description

We theoretically characterized the optimal booking policy in both single-period and multi-period settings, and analyzed the prediction differences between loss-neutral and loss-averse overbooking models.

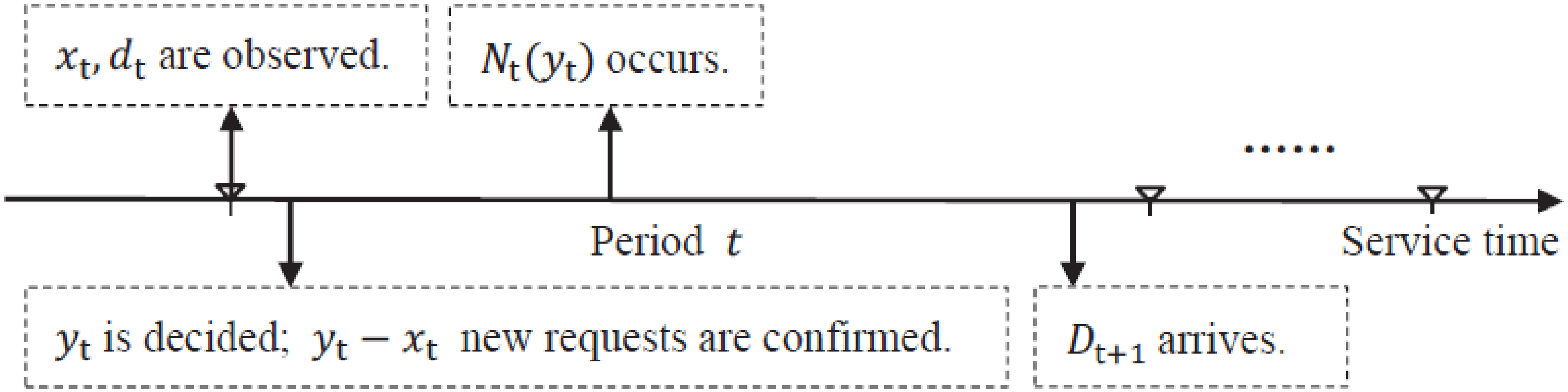
Model Description

- C :capacity of a service system
- x_t :the reservations on hand
- d_t :new reservation requests
- y_t :post-decision reservations on hand.($x_t \le y_t \le x_t + d_t$)
- $N_t(y_t)$:the number of cancellations, a random variable following a Binomial distribution

$$P_{y_t}(n) = P\{N_t(y_t) = n\} = \binom{y_t}{n} q_t^n (1 - q_t)^{y_t - z}$$

- q_t : cancelation probability of a single customer

Sequence of events in period t



Model Description

- f_t : net revenue in period t
- p_t: the fare paid by a customer for reservation in period t
- r_t : refund, with $r_t < p_t$

$$f_t(x_t, y_t) = (y_t - x_t)p_t - N_t(y_t)r_t, \qquad for\ t = 1, \cdots, T.$$

Model Description

- The compensation of denied customers with valid reservations is the only loss incurred due to the overbooking.
- Let x_{T+1} be the number of customers showing up at the time of service. The denied-service cost $c(x_{T+1})$ is then assumed as an increasing convex function of x_{T+1}, which is given by

$$c(x_{T+1}) = \begin{cases} 0, & x_{T+1} \leq C; \\ h(x_{T+1} - C), & x_{T+1} > C. \end{cases}$$

Model Description

- We further assume that the manager is loss-averse and has the following piecewise-linear loss-aversion utility function $u(.)$

$$\mathrm{u}(W) = \begin{cases} W - W_0, & W \geq W_0; \\ \lambda(W - W_0), & W < W_0, \end{cases}$$

- where W_0 is the manager's reference wealth at the beginning of the planning horizon ; W is the manager's post-decision wealth ; λ (≥ 1) is defined as the loss-aversion degree.

3.Single Period Model

This part considers a single period setting in which the dynamics of new reservation requests and customer cancellations over time are absent.

Single Period Model

- Let y be the post-decision reservations on hand. The manager decides on the optimal booking limit y^* by maximizing the expected single-period utility $G(.)$

$$G(y) = E[u(yp - N(y)r) - u(h(y - N(y) - C)^+)]$$

Single Period Model

Theorem 1:

- *(a) G(y) is a concave function in y, hence has an optimal booking limit;*
- *(b) The optimal booking limit y^* decreases in loss-aversion degree λ;*
- *(c) The expected monetary payoff with loss-aversion is lower than the loss-neutral counterpart.*

Example:

- Suppose C=100, q = 0.49, the overbooking compensation cost for each denied customer is \$500, the marginal revenue is p = \$10, and the unit refund is r = \$8.

	λ=1 (loss-neutral)	λ=1.5 (loss-averse)	λ=2 (loss-averse)
booking limit y^*	182	180	178
overbooking pad y^*-C	82	80	78
expected profit	1070	1069	1066
expected utility $G(y^*)$	1070	1057	1050
service level $s_1(y^*)$	0. 0466	0. 0333	0. 0232

Loss-averse managers generally sets low booking limits, and keep better service levels.

4.MultiPeriod Model

This part considers the model of overbooking that accounts for the dynamics of arrivals, cancellations , and decision makings over time.

Model Analysis

- The manager's objective is to find a policy $(y_1, \ldots, y_T)$ to maximize the total expected utility of the reward stream during the whole planning horizon.
- Let $V_{T+1}(x_{T+1}) = u(-c(x_{T+1}))$, and $V_t(x_t, d_t)$ be the maximum total expected utility from period t to the service time.

$$V_t(x_t, d_t) = \max_{x_t \le y_t \le x_t + d_t} G_t(y_t, x_t), \qquad for\ t = 1, \cdots, T$$

Where

$$G_t(y_t, x_t) = E\left[u\big(f_t(y_t, x_t)\big) + V_{t+1}(y_t - N_t(y_t), D_{t+1})\right]$$

Model Analysis

Let $y_t^*(x_t, d_t)$ be the optimal solution with initial reservations on hand x_t and new reservation requests d_t.

$$y_t^*(x_t, d_t) = arg \max_{x_t \leq y_t \leq x_t + d_t} G_t(y_t, x_t)$$

Model Analysis

- $X = \{x_t \mid 0 \le x_t \le x_{max}\}$
- $Y = \{y_t \mid 0 \le y_t \le x_{max} + d_t\}$
- $S = (x_t, y_t) \mid x_t \in X, y_t \in \{x_t + 0, \ldots, x_t + d_t\}$.
- A function $g : S \to R$ is supermodular if

$$g(s') + g(s'') \le g(s' \vee s'') + g(s' \wedge s''), \text{ for all } s' \text{ and } s'' \text{ in } S$$

Model Analysis

Lemma 1. (Topkis (1998))

- *If X and Y are lattices, S is a sublattice of $X \times Y$, S_x is the section of S at x in X, and $g(y, x)$ is supermodular in (y, x) on S, then* $\mathrm{argmax}_{y \in S_x} g(y, x)$ *is increasing in x on $\{x : x \in X, \mathrm{argmax}_{y \in S_x} g(y, x)$ is nonempty$\}$.*

Theorem 1.

- *$G_t(y_t, x_t)$ is supermodular on sublattice S, $t = 1, 2, \ldots, T$.*

Lemma 2.

- *For a given d_t, $y_t^*(x_t, d_t)$ is increasing in x_t, $t = 1, 2, \ldots, T$.*

Assumption 1.

- *For all $t = 1, 2, \ldots, T$, $r_t \le p_t/2$.*

Model Analysis

Theorem 2.

Under the condition of $r_t \le p_t/2$,

- *(a)* $V_t(x_t, d_t)$ *is decreasing and concave in* x_t *for each given* d_t, $t = 1, 2, \ldots, T$;
- *(b) For a given* x_t, $G_t(y_t, x_t)$ *is concave in* y_t, $t = 1, 2, \ldots, T$;
- *(c) The optimal booking policy exhibits a* ***state-dependent booking limit structure****, that is, in each period t, there exists a critical value* $y_t^*(x_t, d_t)$ *such that it is optimal to continue accepting new reservations until the total number of reservations on hand reaches* $y_t^*(x_t, d_t)$.

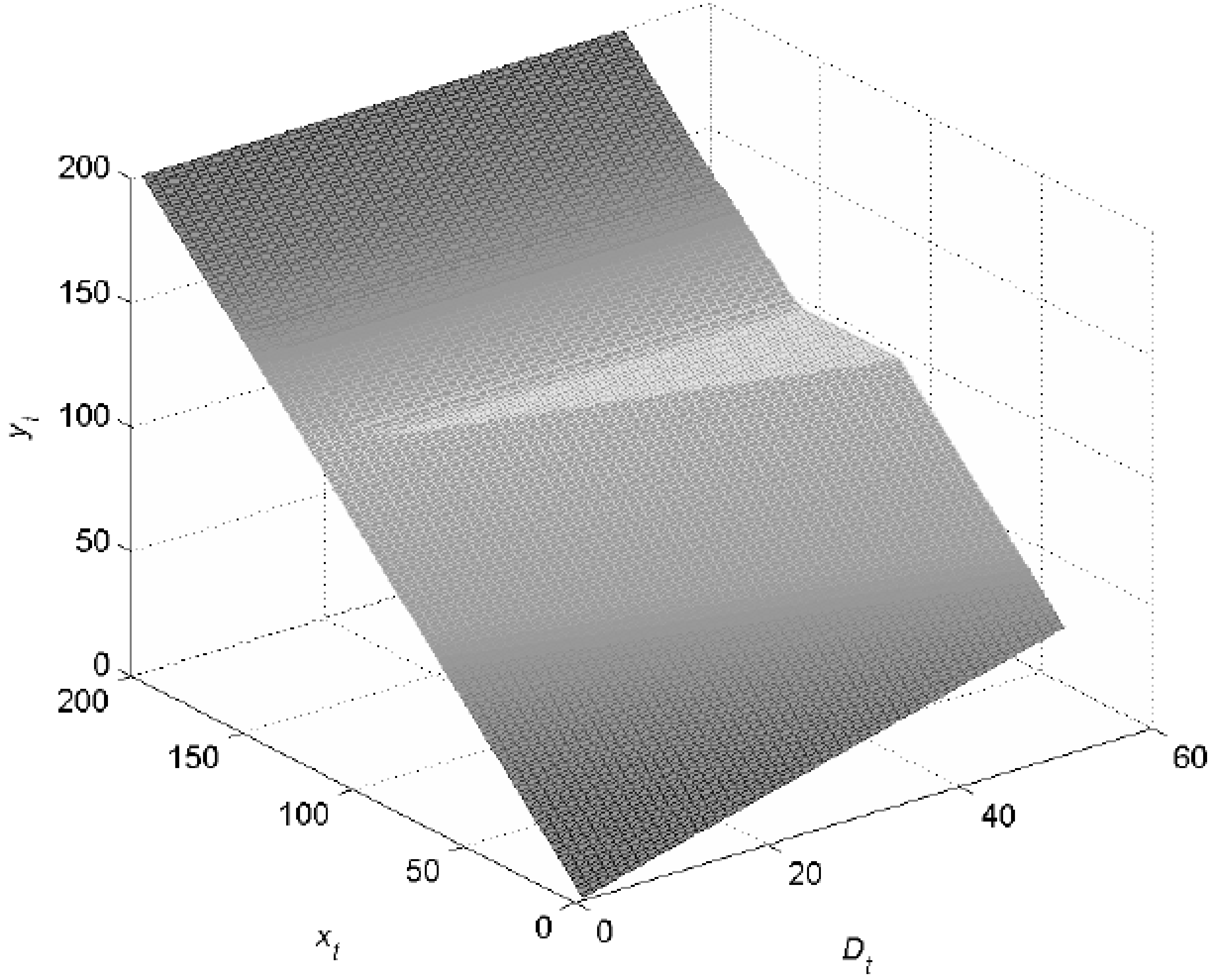
Numerical Examples and Discussions

- $p = 10$, $r = 4$, $q = 0.05$, $T = 20$, $C = 100$, $\lambda = 2$, $D_t \sim$ a modified Poisson distribution with mean $ED_t = 8$, upper bound $D_t \leq 50$, and truncated interval [0, 50]
- The compensation for denied service is described by a quadratic function as:

$$c(x_{T+1}) = \begin{cases} 0, & x_{T+1} \leq C; \\ 20 * (x_{T+1} - C)^2, & x_{T+1} > C. \end{cases}$$

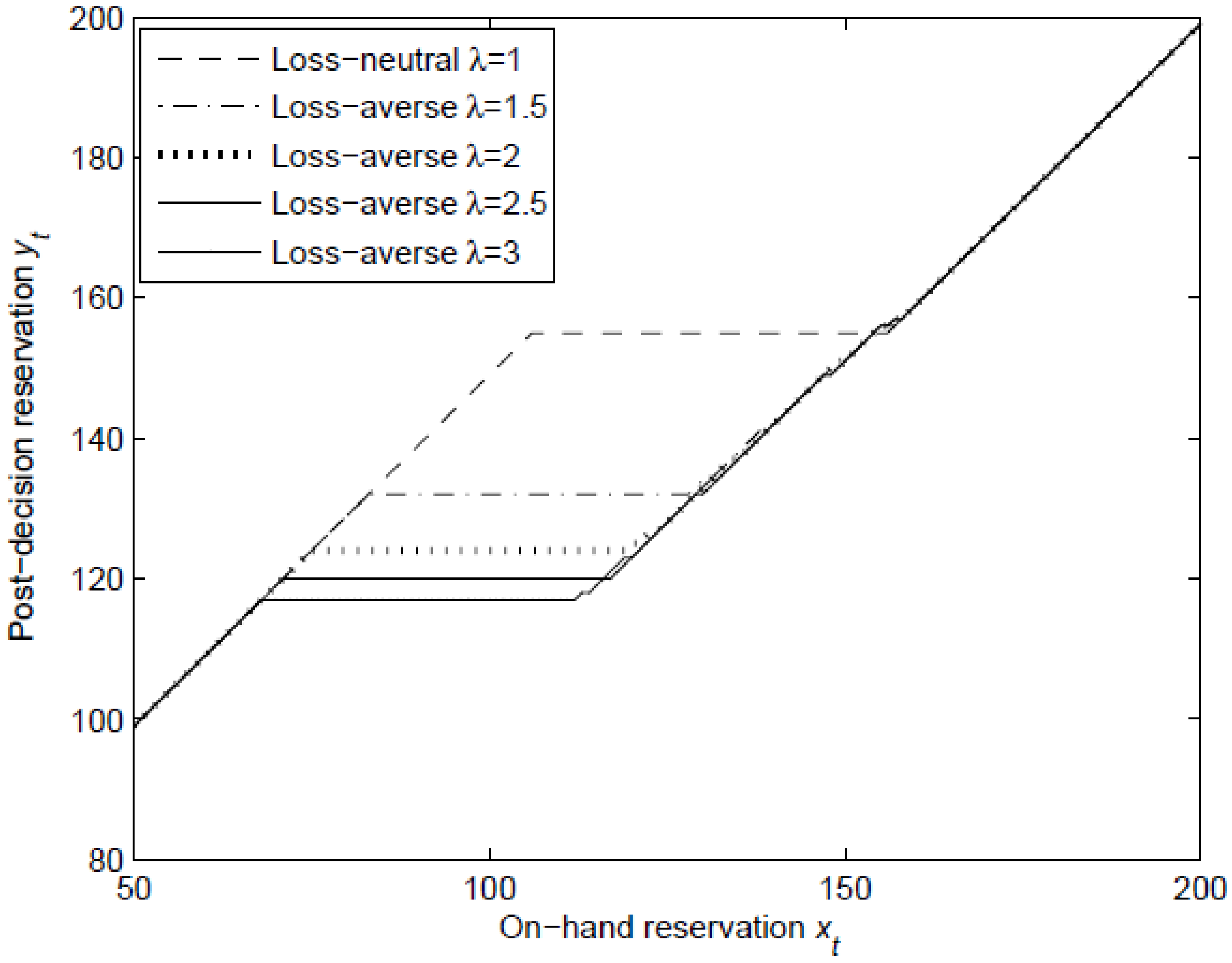
Numerical Examples and Discussions

- As in Figure, all the states at which it is optimal to accept new reservation requests up to a booking limit build a boundary.
- On one side of the boundary, it is optimal to accept all the new reservation requests.
- On the other side of the boundary it is optimal not to accept any new reservation request.



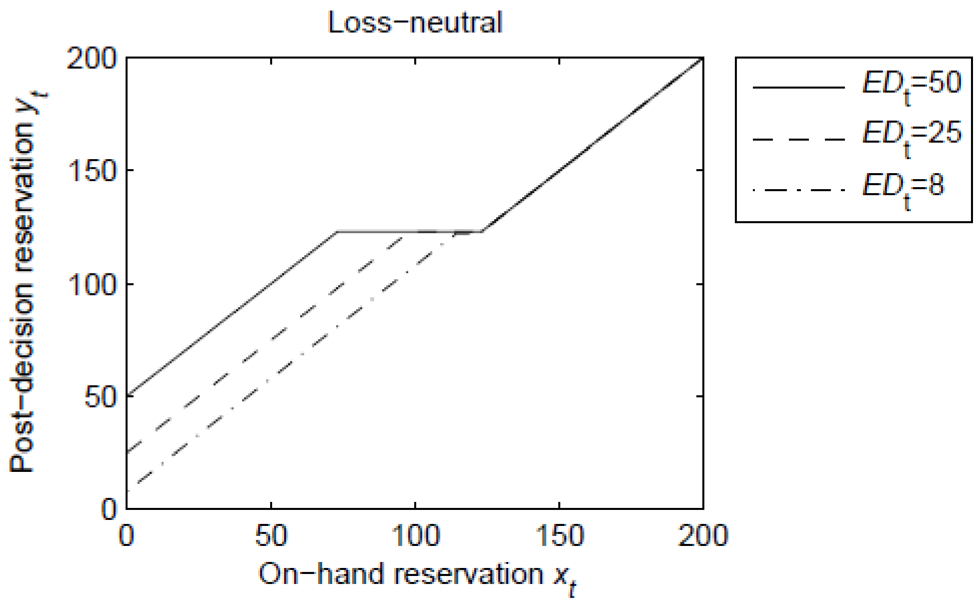
Numerical Examples and Discussions

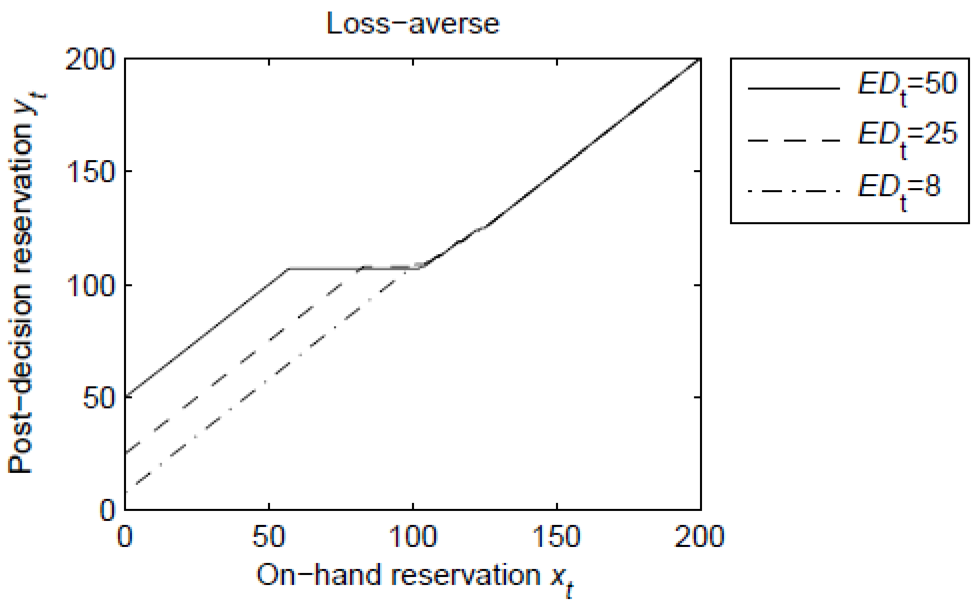
- The degree of loss-aversion is allowed to vary from 1, 1.5, 2, 2.5 to 3.
- It can be seen that the optimal booking limit decreases with the degree of loss-aversion.



Numerical Examples and Discussions

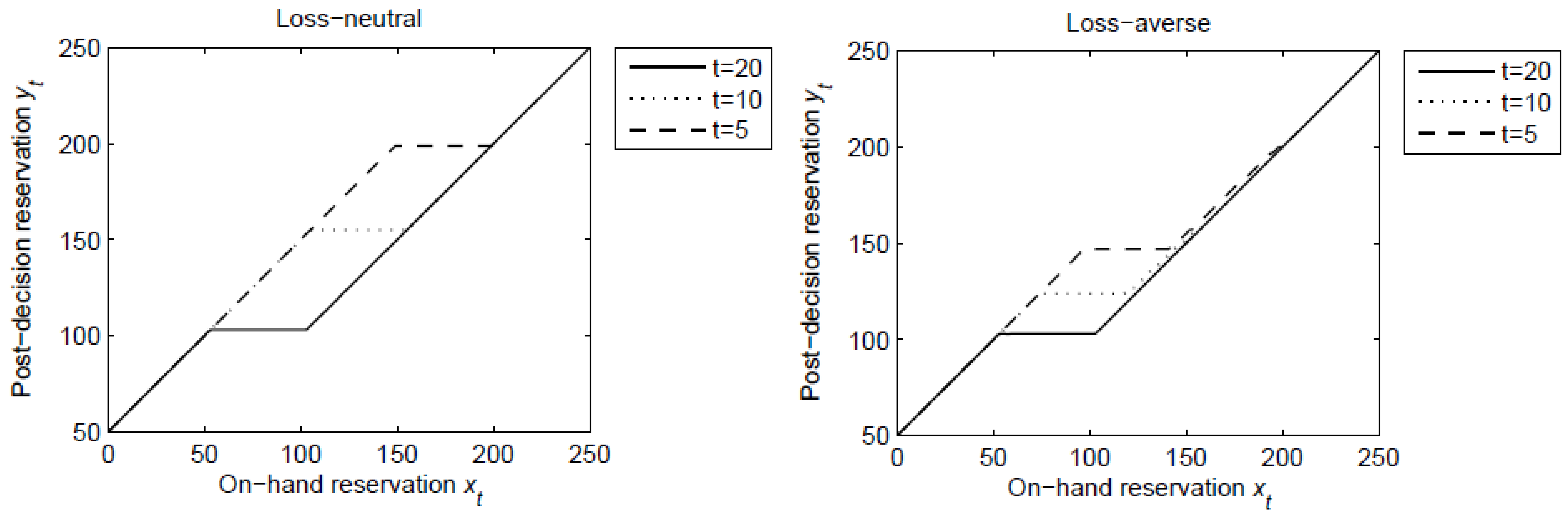
- Figures below intend to illustrate the impact of average new reservation requests on manager's optimal decisions.
- Both loss-neutral and loss-averse managers choose to decline some new requests down to the booking limit when the reservations on hand level is low but ED_t is high.





Numerical Examples and Discussions

- The impact of time left until the service time on the optimal booking limit is then examined.



As shown in the figure, at the beginning of a planning horizon, both managers are likely to seek more revenue through a high booking limit, and then as the time of service approaches, they reduce the booking limit to prevent expensive compensatory payments for possible denied service.

Discussions

- To conclude from all these examples, loss-averse preference induces managers to behave cautiously compared with loss-neutral ones. A loss-averse manager prefers a lower booking limit than a loss-neutral manager, which is in line with the theoretical prediction of the single-period model. Also, a loss-averse manager begins to decline new requests earlier than the counterpart.

5. Conclusions

Conclusions

- We constructed overbooking models considering the loss-aversion behavior.
- We demonstrated that the optimal policy exhibits a booking limit structure in both single-period and multi-period settings.
- We further analyzed the prediction biases between loss-neutral and loss-averse models.
- The results show that loss-averse managers are cautious by preferring lower booking limits and declining new requests earlier than loss-neutral ones.
- We conclude that loss-aversion behavior limits the application of overbooking strategy.

THANKS!

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