CES, Logit, and Rational Inattention

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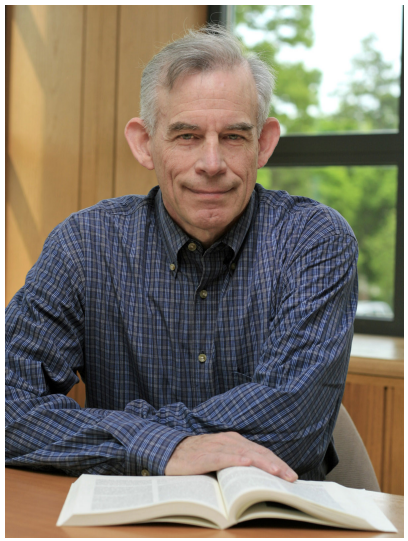
Information & Behavior

- Decision-maker (DM) has an access to all information
- However, the information is costly
- How does it affect behavior?
- What are the market outcomes?

Information Theory and Rational Inattention

- Shannon, 1948
- Sims, 2003
- "...adding information-processing constraints to the kind of dynamic programming problem that is used to model behavior in many current macroeconomic models. It turns out that doing so alters the behavior implied by these models in ways that seem to accord, along several dimensions, with observed macroeconomic behavior."



Logit and CES

Logit

$$P_i(v) = \frac{e^{v_i/\mu}}{\sum_{j=1}^{N} e^{v_j/\mu}}$$

CES utility function

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{1/\rho}$$

Random Utility Model

$$U_i = v_i + \mu \varepsilon_i$$

Problem of Rationally Inattentive DM 1

1) The DM chooses among N possible actions, is endowed with prior about their payoffs,
2) chooses what additional information to process,
3) receives signals of the selected form,
4) selects one action.

- Action set: $A = \{1, \cdots, N\}$
- State: $\mathbf{v} \in \mathbb{R}^N$ where v_i is the payoff of action $i \in A$
- The DM is a Bayesian expected utility maximizer

Problem of Rationally Inattentive DM 2

The agent's problem is to find an information processing strategy (modeled as a joint distribution between v and the chosen product i) maximizing

$$\mathbb{E}v_i - \lambda\kappa.$$

(utility: expected payoff less cost of information).
Entropy-based cost function (reduction of uncertainty):

$$\hat{c}(F) \equiv \lambda\Big(H(G) - E_{\mathbf{s}}[H(F(\cdot|\mathbf{s}))]\Big),$$

where $\lambda \geq 0$, $H(B)$ is entropy of B, measure of uncertainty, e.g.

$$H(B) = -\sum_k P_k \log(P_k).$$

Description of our Model

- N goods, $i = 1, ..., N$ – perfect substitutes
- each good has price p_i
- utility function of decision maker $u_i = \ln q_i$,
- income y
- indirect utility function $v_i = \ln\left(\frac{y}{p_i}\right)$

RI

- Observing prices is costly
- DM is rationally inattentive

Matějka and McKay (2015) derive:

$$P_i(v) = \frac{P_i^0 e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{N} P_j^0 e^{\frac{v_j}{\lambda}}}, \quad i = 1, ..., N.$$

RI and CES

Conditional expected demand:

$$D_i = \frac{P_i^0 p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, ..., N.$$

The same demand system from CES:

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{\frac{1}{\rho}},$$

where $\rho = 1/(\lambda + 1)$, and the coefficients β_j depend positively on the corresponding unconditional probabilities P_j^0.

Implications

- If $\lambda \to 0$ then $CES \sim \max$
- If $\lambda \to \infty$ then $CES \sim Cobb - Douglas$
- If $\lambda \nearrow$ then mark-up $1 - \rho \nearrow$

Conclusion

- We endogenize elasticity of substitution and coefficients of CES function
- We connect theory of rational inattention with popular neoclassical economics model
- Such connection is important for policy implications and empirical analysis

Thank you for your attention